



# INSTITUTE OF MATHEMATICS EDUCATION

## Junior Maths Olympiad 2026 (Higher Level)

Std.: VII and VIII

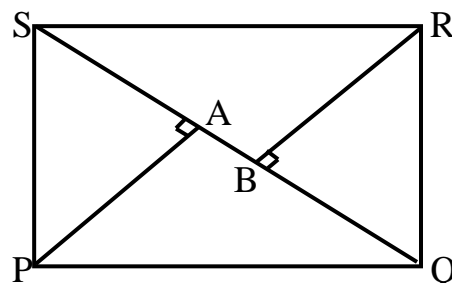
Question paper

Date: 01.02.2026

Time: 2 Hours

Total Marks: 100

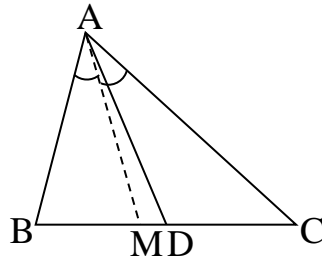
- Q.1.** Find the domain of the function  $f(x) = \sqrt{\frac{2[x]-7}{11-[x]}}$ , where  $[x]$  is greatest integer less than or equal to  $x$ . (6 marks)
- Q.2.** An A.P. consists of positive integers. The sum of its first 12 terms is between 495 and 500. If it's 6<sup>th</sup> term is 38, then find it's 10<sup>th</sup> term. (6 marks)
- Q.3.** Consider  $\Delta PQR$ . The medians  $PS$  &  $RT$  intersect at right angles at  $G$ . If  $QR = 3$  cm, and  $PQ = 4$  cm, then find the value of  $(PR)^2$ . (6 marks)
- Q.4.** The 4-digit numbers are formed by using the digits 3, 5, 7, and 9 where repetition of the digits is allowed. These numbers are then written in descending order by assigning serial numbers to them. For example, the number 9999 has serial number 1. Find the serial number of the number 5373. (6 marks)
- Q.5.** Find the sum of all real values of  $x$  satisfying the following equation: (6 marks)  
 $(x^2 - 11x + 29)^{(x^2 + 2x - 48)} = 1$
- Q.6.** Find minimum value of (i)  $2x^2 + \frac{1}{x^2} - 2x + 15$  if  $x \in \mathbb{R}$  (5 marks)  
(ii)  $x^2 + \frac{9}{x^2 + 1}$  if  $x \in \mathbb{R}$  (3 marks)
- Q.7.** Let  $A = \{n \in \mathbb{N} : \text{HCF}(n, 15) = 1\}$  and  $B = \{x \in 2k : k \in \{1, 2, 3, \dots, 100\}\}$ .  
Then a) Find number of elements in  $A \cap B$  (5 marks)  
b) Find the sum of elements in  $A \cap B$  (3 marks)
- Q.8.** Find the remainder if  $15! + 14! - 13!$  is divided by 17. (8 marks)
- Q.9.** Let  $p, q, r$  be in an Arithmetic Progression. Let the centroid of the triangle with vertices  $(p, r), (p, q), (-5, q)$  be  $\left(\frac{7}{3}, \frac{14}{3}\right)$ . Let  $\alpha$  and  $\beta$  be the roots of the equation  $px^2 + qx - r = 0$ . Find the sum of  $\alpha$  and  $\beta$ . (8 marks)
- Q.10.** Refer figure. PQRS is a rectangle. Points A & B are on QS such that  $PA \perp SQ$  and  $RB \perp SQ$ .  
(i) Prove that  $AQ^2 + QB^2 = SA^2 + SB^2$  (4 marks)  
(ii) If  $PQ = 8$  cm and  $PS = 6$  cm, then find the length of AB. (4 marks)



**Q.11.** Find the number of ways to write 2010 in the form  $1000a + 100b + 10c + d$ , where  $a, b, c, d$  are integers from 0 to 99.

[For example,  $2010 = (1000 \times 1) + (100 \times 7) + (10 \times 30) + 10$ ]. **(10 marks)**

**Q.12.** Refer figure. In  $\triangle ABC$ ,  $BC = a$ ,  $CA = b$ ,  $AB = c$  &  $A, B, C$  denote angles **(10 marks)**



(i) Prove that the length of the median  $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

(ii) Also find the length of the median through B if  $a = 10$ ,  $b = 7$  and  $c = 6$

(iii) Prove that the length of the angle bisector  $AM = \frac{2bc}{b+c} \cos \frac{A}{2}$

$$\left( \text{Given } \sin \theta = 2 \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{\theta}{2} \right) \right)$$

**Q.13.** (i) Find the number of positive divisors of:  $(2024)^3 + [3(2024)(2025) + 1]$  **(7 marks)**

(ii) How many of these divisors are of the form  $4n + 1$ ,  $n \in \mathbb{N}$ ? **(3 marks)**

